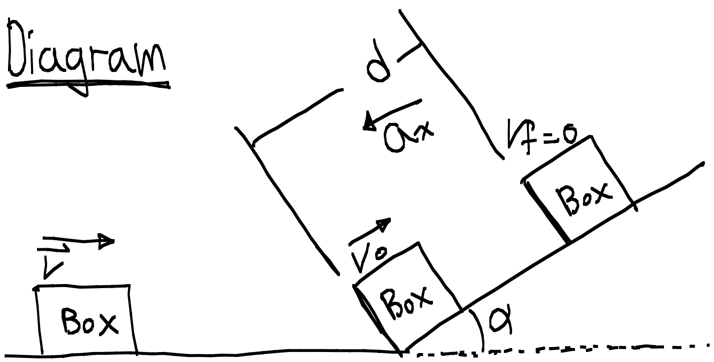
# A Work-Energy Practice Problem

Note Title 10/18/2007

Diagram

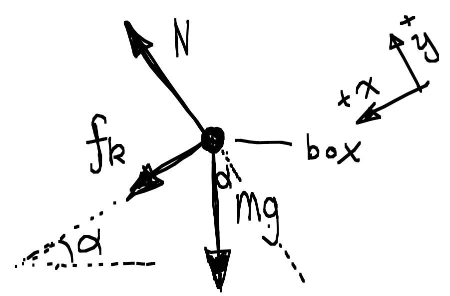



A box was initially pushed and push force was removed. The box ~~starts~~ up the inclined plane with velocity of $V_0$ and travels distance d to stop its motion

Given
$V_0$ = initial velocity after block is on the inclined plane.
$V_f = 0$ m/s
$\alpha$ = angle incline with horizontal
$\mu_k$ = given kinetic friction constant
$a_x = ?$

Part I Goal = Find the acceleration of the box

## Force Diagram




## Equations

$F_{net\,y} = N - mg\cos\alpha$

↳ $F_{net\,y} = 0$ (because $a_y = 0$)

then $N = mg\cos\alpha$

$F_{net\,x} = f_k + mg\sin\alpha$

$F_{net\,x} = ma_x$ — $f_k = N\mu_k$ — $N = mg\cos\alpha$

$ma_x = mg\mu_k\cos\alpha + mg\sin\alpha$

$\cancel{m}a_x = \cancel{m}g(\mu_k\cos\alpha + \sin\alpha)$

$a_x = g(\mu_k\cos\alpha + \sin\alpha)$

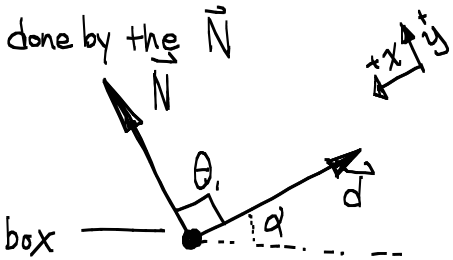
## Checks

- result is + since acceleration is down plane. ✓
- $a_x = m/s^2$ ✓
- if $\alpha = 0$, then $a_x = \mu_k g$ ✓
- if $\alpha = 90°$, then $a_x = g$ (free fall, no friction)

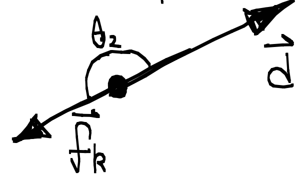
**Part II Goal** = Find total work done on to the box

## Work done by each force.

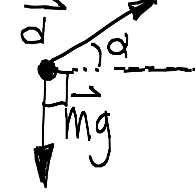
Work done by the $\vec{N}$




Work done by the $\vec{f_k}$




Work done by the $\vec{mg}$




$\theta_3 = 90° + \alpha$

※ $\theta$ is used to distinguish the difference between the angle from $\alpha$

※ Work = (Magnitude of Force) x (Magnitude of displacement) x (direction of displacement)

$$W_N = Nd\cos\theta_1 = Nd(0) = 0$$

$$W_{fk} = f_k d\cos\theta_2 = f_k d(-1) = -f_k d = -(N\mu_k)d = -(mg\mu_k\cos\alpha)d$$

$$W_{mg} = mgd\cos(\theta_3) = mgd\cos(90° + \alpha) = -mgd\sin(\alpha)$$

## Find net (total) work

$$W_{net} = W_n + W_{fk} + W_{mg}$$

$$W_{net} = 0 - mg\mu_k d\cos\alpha - mgd\sin\alpha)$$

$$W_{net} = -mgd(\mu_k\cos\alpha + \sin\alpha).$$

Part III Goal = Find $V_0$ in terms of $\mu_k, g, \alpha, d$. (hint; use energy work theorem)

Equation

Energy work theorem

$W_{net} = \Delta K$ ※ $K = \frac{1}{2}mv^2$

↳ $\Delta K = K_f - K_i$

$K_f = \frac{1}{2}mv_f^2$ ↲ ↳ $K_i = \frac{1}{2}mv_0^2$

$W_{net} = \Delta K = \frac{1}{2}m(\cancel{v_f^2}^{\,0} - v_0^2)$ ※ $v_f = 0\,m/s$ from given

$-\cancel{m}gd(\mu_k\cos\alpha + \sin\alpha) = -\frac{1}{2}\cancel{m}v_0^2$

$2gd(\mu_k\cos\alpha + \sin\alpha) = V_0^2$

$V_0 = \pm\sqrt{2gd(\mu_k\cos\alpha + \sin\alpha)}$

$V_0 = -\sqrt{2gd(\mu_k\cos\alpha + \sin\alpha)}$ ※ take − root because down slope was set to be +

Checks

- expression under $\sqrt{\ }$ is + so result is real. ✓
- $V_0 = \sqrt{m/s^2 \cdot m}$ ✓
- if $\theta = 0°$ then $V_0 = -\sqrt{2gd\mu_k}$
- if $\mu_k = 0$ then $V_0 = 0$
- if $\theta = 90°$ then $V_0 = -\sqrt{2gd}$ (like an object thrown vertically)