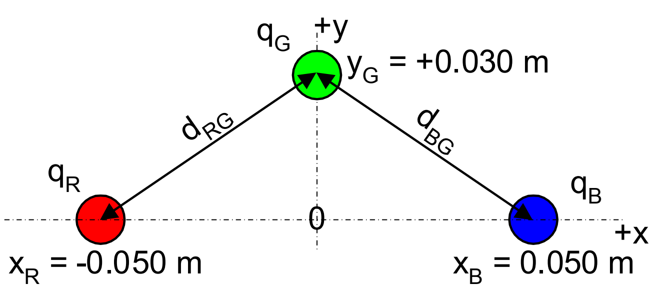
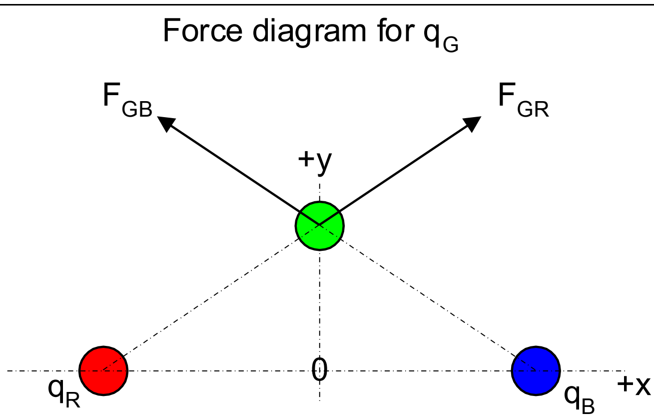
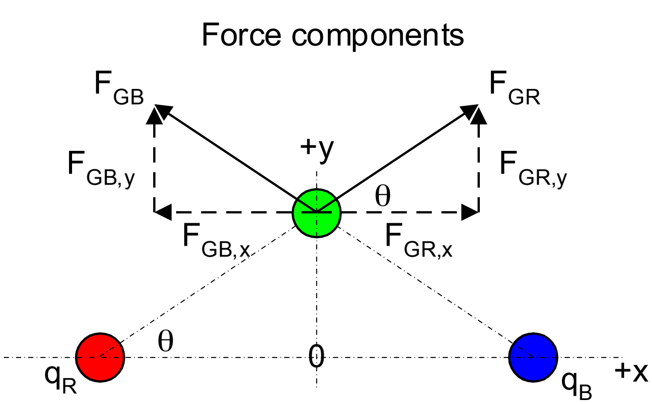
Finding the net electric force on a charge due to two other charges (2-dimensional symmetric arrangement)

Refer to the diagram below. Charges $q_R = +\ 4.0\ \mu C$ and $q_B = +\ 4.0\ \mu C$ are positioned as shown. A green charge $q_B = +\ 1.0\ \mu C$ is positioned on the perpendicular bisector of the line joining the red and blue charges. What are the magnitude and direction of the net, electric force on the green charge?

<table>
<tr><th>Solution</th><th>Notes</th></tr>
<tr><td>
</td><td>x- and y-coordinate axes are selected to take advantage of the symmetry of the situation. We will see that with this selection, the x-component of the net force will be 0, and the y-component of the net force will be twice that of the y-component due to either the red or blue charge.

The positions x_R, x_B, y_G, are given with respect to the defined axes. These may be positive or negative numbers. The distances, d_{RG} and d_{BG}, on the other hand, are magnitudes. They are calculated using the Pythagorean theorem.</td></tr>
<tr><td>
</td><td>The force diagram is drawn. The forces extend from a single point at the position of the green charge. Since the red and blue charges are equal and are equidistant from the green charge, we know that the magnitudes of the forces due to the red and blue charges on the green charge will be equal.</td></tr>
<tr><td>
</td><td>The two forces are resolved into their components. Now we can see how the problem is simplified by the symmetry of the arrangement of charges (the green charge is equidistant from the red and blue charges) and the fact that we selected the y-axis to be the perpendicular bisector of the line joining the charges, the origin being centered on that line. With this arrangement, $F_{GB,x} = -\ F_{GR,x}$; therefore, the x-component of the net force on the green charge is 0.

The y-components of the two forces are equal so we can say:
$$F_{net,G,y} = 2F_{GR,y}$$
We are left to calculate only a single force, $F_{GR,y}$.</td></tr>
</table>

$$F_{GR,y} = \lvert F_{GR} \rvert \sin\theta$$ $$= k\frac{\lvert q_G \rvert \lvert q_R \rvert}{d_{RG}^{\ 2}}\frac{y_G}{d_{RG}}$$	The y component of the force of the red charge on the green charge is the magnitude of F_{GR} multiplied by the sine of the angle, θ. Note that this angle is the angle that the x-axis makes with the line joining the red charge to the green charge. The side opposite θ is y_G, and the hypotenuse is d_{RG}. Thus we substitute the ratio y_G / d_{RG} for $\sin\theta$. We also substitute Coulomb's Law with the charges in absolute signs.
$$F_{GR,y} = k \lvert q_G \rvert \lvert q_R \rvert \frac{y_G}{d_{RG}^{\ 3}}$$ $$= k \lvert q_G \rvert \lvert q_R \rvert \frac{y_G}{\left(\sqrt{\lvert y_G \rvert^2 + \lvert x_R \rvert^2}\right)^3}$$ $$= k \lvert q_G \rvert \lvert q_R \rvert \frac{y_G}{\left(\lvert y_G \rvert^2 + \lvert x_R \rvert^2\right)^{3/2}}$$	We collect the terms in d_{RG} and apply the Pythagorean theorem to d_{RG}. Note that the positions under the square root sign are placed between absolute signs, since we're interested in distances here. Of course, since the values are squared, the result will be positive in any case. The y_G in the numerator is not in absolute value signs, because the sign of y_G determines whether the force component is in the +y or – y direction.
$$F_{net,G,y} = 2F_{GR,y}$$ $$= 2k \lvert q_G \rvert \lvert q_R \rvert \frac{y_G}{(\lvert y_G \rvert^2 + \lvert x_R \rvert^2)^{3/2}}$$	We double the previous result to get the y-component of the net force on the green charge. In making substitutions below, note the conversion of microcoulombs to coulombs.
$$F_{net,G,y} = \frac{2k \lvert q_G \rvert \lvert q_R \rvert y_G}{(\lvert y_G \rvert^2 + \lvert x_R \rvert^2)^{3/2}}$$ $$= \frac{2(8.99E9\, Nm^2/C^2)(1.0\,E-6\,C)(4.0E-6\ C)(0.030\,m)}{[(0.030\,m)^2 + (0.050\,m)^2]^{3/2}}$$ $$= 10.9\,N$$	

The y-component of the net force on the green charge is +10.9 N, and the x-component is 0; therefore, the magnitude of the net force is 10.9 N and the direction is 90°.