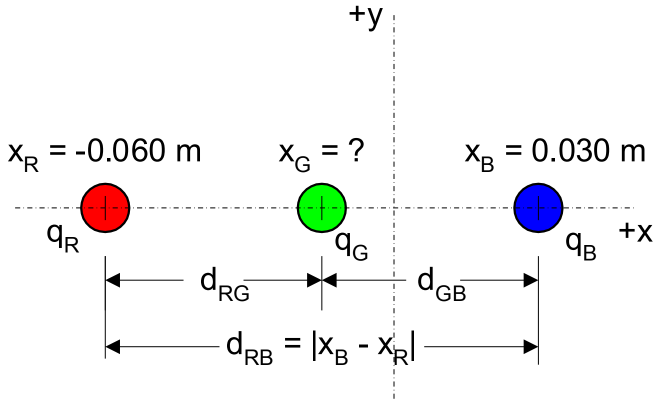
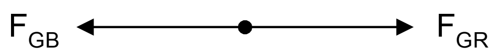
Finding the position between two positive charges where the net force on a third, positive charge is 0

Refer to the diagram below. Charges $q_R = + 1.0\ \mu C$ and $q_B = + 4.0\ \mu C$ are positioned as shown. What must the position of the positive green charge be so that the net electric force on it is 0?

<table>
<tr><th>Solution</th><th>Notes</th></tr>
<tr><td>


Force diagram for q_G



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The positions x_R, x_B, x_G, are given with respect to the defined axes. These may be positive or negative numbers.

The distances between charges, denoted by the symbol d with double subscripts, is a magnitude. Thus, for example, $d_{RB} = |x_B - x_R| = |0.030\text{ m} - (-0.060\text{ m})| = 0.090$ m.

The force diagram is drawn. Like always, the forces extend from a single point representing the green charge. The symbol F_{GB} means force of the blue charge on the green charge.
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$$F_{net,G} = |F_{GR}| - |F_{BG}|$$
$$0 = k\frac{|q_G||q_R|}{d_{RG}^{\ 2}} - k\frac{|q_B||q_G|}{d_{GB}^{\ 2}}$$
$$= k|q_G|\left(\frac{|q_R|}{d_{RG}^{\ 2}} - \frac{|q_B|}{d_{GB}^{\ 2}}\right)$$

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The net force equation is written. Note that the forces on the right are written as magnitudes as is our usual practice in net force problems. The negative sign is explicitly written to indicate the direction of the force.

Zero is substituted for the net force. Coulomb's Law is used for the two force terms. The charges are placed in absolute value signs.
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$$\frac{q_B}{d_{GB}^{\ 2}} = \frac{q_R}{d_{RG}^{\ 2}}$$
$$\frac{d_{GB}^{\ 2}}{d_{RG}^{\ 2}} = \frac{q_B}{q_R}$$
$$\frac{d_{GB}}{d_{RG}} = \sqrt{\frac{q_B}{q_R}}$$

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The quantity in parentheses above is set equal to 0, and the ratio of the distances is solved for.

The absolute value signs around the q's are dropped, since we know that all the charges are positive.

The positive root is kept, since we know a ratio of distances must be positive.
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$$d_{GB} = d_{RB} - d_{RG}$$

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In the algebraic solution below, we'll make use of this relationship between the distances. This will allow us to get rid of one unknown.
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$d_{GB} = d_{RG}\sqrt{\frac{q_B}{q_R}}$ $d_{RB} - d_{RG} = d_{RG}\sqrt{\frac{q_B}{q_R}}$ $d_{RB} = d_{RG}\left(1+\sqrt{\frac{q_B}{q_R}}\right)$ $d_{RG} = d_{RB}\left(1+\sqrt{\frac{q_B}{q_R}}\right)^{-1}$	We eliminate the unknown d_{GB} and solve for d_{RG} in terms of the known distance d_{RB} and the two known charges.
$d_{RG} = d_{RB}\left(1+\sqrt{\frac{q_B}{q_R}}\right)^{-1}$ $= (0.090\,m)\left(1+\sqrt{\frac{4.0\,\mu C}{1.0\,\mu C}}\right)^{-1}$ $= 0.030\,m$	Substituting known values gives the result for d_{RG}.

To complete the problem, we must solve for x_G, the position of the green charge. Since x_R = -0.060 m and the green charge is 0.030 m to the right of the red charge, the position of the green charge is -0.030 m.